Algebra I Review for Incoming Algebra II / Algebra II Honors Students

Solve each equation.

1. \( 5x + 8 = 3 + 2(3x - 4) \)
2. \( -5(2n - 3) = 7(3 - n) \)
3. Victoria goes to the mall with $60. She purchases a skirt for $12 and perfume for $35.99. She also spends $3.25 on food. She still wants to buy a belt. How much money can she spend on the belt?
4. Nicole makes $9.50 per hour working at an electronics company. She plans to buy a hand-held computer, the least expensive of which costs $245.60 and the most expensive of which costs $368.40. Write and solve an inequality describing how long Nicole will have to work to be able to buy a hand-held computer.

Solve each inequality.

5. \( 3(-w - 6) < 2(2w + 8) + 1 \)

Solve each compound inequality. Then graph the solution set.

6. \( \frac{w}{3} < 1 \) or \( 3w + 5 > 11 \)

7. \( 2 + 3x > 8 \) or \( 4 - 7x \leq -17 \)

8. Write an equation and state the slope for the line that passes through \((9, 22)\) and \((15, 36)\).

9. Write the point-slope form, slope-intercept form, and standard form of an equation for a line that passes through \((-1, 2)\) with slope 4.

10. Determine whether \( y = 4x + 5 \) and \( y = \frac{1}{4}x - 2 \) are perpendicular. Explain.

11. Write an equation of the line that is parallel to the graph of \( y = -4x + 2 \) and passes through \((2, -4)\).

12. To fill two new aquariums, Laura bought some saltwater fish for $2 each and some freshwater fish for $1 each. If she bought a total of 15 fish and spent a total of $23, how many fish of each kind did she buy?

13. Two trains \( A \) and \( B \) are 240 miles apart. Both start at the same time and travel toward each other. They meet 3 hours later. The speed of train \( A \) is 20 miles faster than train \( B \). Find the speed of each train.

14. Scott bought a pen and received change of $4.75 in 25 coins, all dimes and quarters. How many of each kind did he receive?

15. Five times one number added to another number is 32. Three times the first number minus the other number is 8. Find the numbers.
16. The graphs of \(2x + 3y = 5\) and \(3x + y = 18\) contain two of the sides of a triangle. A vertex of the triangle is at the intersection of the graphs. What are the coordinates of the vertex?

17. Use the rate formula to write an equation for the distance traveled by a boat upstream against a current and another equation for the distance traveled by a boat downstream with the current, where \(r = \text{speed of the boat, and } c = \text{rate of the current, and } s = \text{total speed.}\) Then solve each equation for the time.

18. A boat travels 33 miles downstream in 4 hours. The return trip takes the boat 7 hours. Find the speed of the boat in still water.

19. Laura can weed the garden in 1 hour 20 minutes and her husband can weed it in 1 hour 30 minutes. How long will they take to weed the garden together?

20. The inlet pipe of an oil tank can fill the tank in 1 hour 30 minutes. The outlet pipe can empty the tank in 1 hour. How long it will take to empty a full tank if both pipes are open?

Solve the system of inequalities by graphing.

21. \(y > x + 2\)
\(y \leq -2x - 1\)

Determine the best method to solve each system of equations. Then solve the system.

22. \(x = 2y - 1\)
\(3x + y = 11\)

23. \(5x - y = 17\)
\(3x - y = 13\)

The length of a rectangular garden ABCD is 9 feet more than its width. It is surrounded by a brick walkway 4 feet wide as shown below. Suppose the total area of the walkway is 400 square feet.

24. Write a polynomial to represent the length of \(PQ\).

25. What are the dimensions of the garden?
The measures of two sides of a triangle are given. If $P$ is the perimeter, find the measure of the third side.

26. $P = 8x^2 + 4x - 1$

Simplify. Assume that no denominator is equal to zero.

27. $(3a^2b^5)(-2ab^3)$
28. $4a^4b^8 + 2(ab^5)^4 + 4(a^5b^4)^2$
29. $\frac{4a^{-3}d^2}{8a^2d^{-3}}$
30. $\frac{(3r^2t^5)^3}{(-3r^2t^7)^2}$

Evaluate each product or quotient. Express the results in both scientific notation and standard form.

31. $\frac{1.6 \times 10^3}{8 \times 10^7}$
32. Find the degree of the polynomial $2x^2y - 4x^5 + 6xy^3$.
33. Write $3x^2 - x - 3 + x^3$ in standard form. Identify the leading coefficient.

Find each product.

34. $3x^3y(2x^2y - 5xy^2 + 8y^3x^2)$
35. $(2n + 3)(3n^2 - 4n + 1)$
36. $(5y + 6)^2$
Factor each polynomial, if possible. If the polynomial cannot be factored, write prime.

37. $10x^2yz - 22x^3y^2z$
38. $2xy - 4x + 3y - 6$
39. $m^2 + 12m - 28$
40. $5t^2 + 17t - 12$
41. $6p^2 - 20p + 16$
42. $49a^2 - 169$
43. $3x^5 - 75x^3$
44. $81c^2 + 72c + 16$
45. $25x^2 + 70x - 49$

Solve each equation. Check the solutions.

46. $12b^2 - 8b = 0$
47. $y^2 + 4y = 45$
48. $9n^2 + 6n = 3$

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

49. $12v^2 - 6 = -v$
50. $d^2 - 14d - 22 = 0$
51. $15n^2 - 3 = 4n$

State the value of the discriminant for each equation. Then determine the number of real roots of the equation.

52. $9a^2 = 6a - 1$
53. Find two consecutive even integers $x$ and $x + 2$ with a product of 80.
54. The roots of a quadratic equation are $-2$ and $2$. The maximum point of the graph of its related function is at $(0, 4)$. Sketch the graph.
55. Use the graph to find where the roots of the function lie and the minimum point of the graph of the related function.

56. Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of \( y = -2x^2 + 4x - 5 \). Identify the vertex as a maximum or a minimum.

**Simplify each expression.**

57. \( \sqrt{50x^3y^2} \)

58. \( \frac{5\sqrt{2}}{\sqrt{10} - 3} \)

59. \( 2\sqrt{24} + \sqrt{54} + 3\sqrt{150} \)

60. \( \left( \sqrt{11} - \sqrt{6} \right) \left( \sqrt{2} + \sqrt{33} \right) \)

61. The Distance Formula \( d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \) can be used to find the distance between two points \((x_1, y_1)\) and \((x_2, y_2)\). Find the distance between \((6, 0)\) and \((-5, 4)\).

62. The points \( A\left(-3, b\right) \) and \( B\left(1, 3\right) \) are 5 units apart. Find the value of \( b \).

63. Find the area and the perimeter of a square with a side of length \( 2\sqrt{3} + 6\sqrt{2} \) inches.

64. The length of the base of a right-angled triangle \( ABC \) is 6 centimeters and the length of the hypotenuse is 10 centimeters. Find the area of the triangle.
65. Triangle $ABC \sim \triangle LMN$. Find the value of $x$.

\[ \begin{align*}
\triangle ABC & \sim \triangle LMN \\
\frac{6}{10} &= \frac{x}{15} \\
x &= \frac{6 \times 15}{10} \\
x &= 9
\end{align*} \]

66. Find the coordinates of the midpoint of the segment with endpoints $(7, -1)$ and $(-1, 5)$.

\[ \left( \frac{7 + (-1)}{2}, \frac{-1 + 5}{2} \right) = (3, 2) \]

67. State the excluded values of $\frac{x^2 + x - 12}{x^2 - 7x + 12}$.

The denominator factors into $(x - 4)(x - 3)$, so the excluded values are $x = 4$ and $x = 3$.

68. Simplify $\frac{k^2 + 2k - 15}{k^2 - 4k + 3}$. State the excluded values of $k$.

\[ \frac{(k - 5)(k + 3)}{(k - 3)(k - 1)} \]

The excluded values are $k = 3$ and $k = 1$.

**Find each product.**

69. \[ \frac{y^2 - 9}{4} \cdot \frac{8}{y + 3} \]

\[ \frac{(y - 3)(y + 3)}{4} \cdot \frac{8}{y + 3} = \frac{2(y - 3)}{2} = y - 3 \]

70. \[ \frac{r^2 + 2r - 3}{r^2 + 5r + 6} \cdot \frac{r + 2}{r^2 - 1} \]

\[ \frac{(r + 3)(r - 1)}{(r + 3)(r + 2)} \cdot \frac{r + 2}{(r + 1)(r - 1)} = \frac{1}{r + 1} \]

**Find each quotient.**

71. \[ \frac{2x}{x + 7} \div \frac{8x^2}{x^2 + 8x + 7} \]

\[ \frac{(x + 7)(x + 1)}{x + 7} \div \frac{8x}{(x + 1)(x + 7)} = \frac{x + 1}{8x} \]

72. \[ \frac{x^2 + 5x - 14}{x^2 - 2x - 15} \div \frac{7x - 14}{6x + 18} \]

\[ \frac{(x - 2)(x + 7)}{(x + 3)(x - 5)} \div \frac{7(x - 2)}{6(x + 3)} = \frac{x - 2}{x - 5} \]

73. \( (12a^4b^2 - 5a^2b^3 - 15a^3b^2) + (3a^2b) \)

**Find each sum.**

74. \[ \frac{2r - 3}{r - 5} + \frac{6r + 7}{r - 5} \]

\[ \frac{2r - 3 + 6r + 7}{r - 5} = \frac{8r + 4}{r - 5} \]
75. \[ \frac{3y - 6}{y^2 - 4y + 4} + \frac{1}{y - 2} \]

76. \[ \frac{-2}{6 - n} + \frac{13}{n^2 - 36} \]

Find each difference.

77. \[ \frac{25n^2}{5n - 4} - \frac{16}{4 - 5n} \]

78. \[ \frac{-9}{5y - 35} - \frac{-4}{y^2 - 7y} \]

Solve. State any extraneous solutions.

79. \[ \frac{2}{m + 1} - \frac{1}{3m + 3} = \frac{-5}{9} \]

80. \[ \frac{x^2}{x - 3} - \frac{9}{x - 3} = -2 \]

81. Solve \(|x - 4| = 8\).

82. Graph \(f(x) = |x + 4|\).

83. Solve \(|d + 1| > 8\).

84. Solve \(|2x - 3| \leq 9\).
Answers:

Example for 1 – 2.

**EXAMPLE 2** Solve an Equation with Grouping Symbols

Solve $6(5m - 3) = \frac{1}{3}(24m + 12)$.

$6(5m - 3) = \frac{1}{3}(24m + 12)$ \hspace{1cm} Original equation

$30m - 18 = 8m + 4$ \hspace{1cm} Distributive Property

$30m - 18 - 8m = 8m + 4 - 8m$ \hspace{1cm} Subtract $8m$ from each side.

$22m - 18 = 4$ \hspace{1cm} Simplify.

$22m - 18 + 18 = 4 + 18$ \hspace{1cm} Add 18 to each side.

$22m = 22$ \hspace{1cm} Simplify.

$m = \frac{22}{22}$ \hspace{1cm} Divide each side by 22.

$m = 1$ \hspace{1cm} Simplify.

1. $x = 13$

2. $n = -2$

3. Victoria can spend no more than $8.76 on the belt.

$12 + 35.99 + 3.25 + x \leq 60$

Solve the inequality by subtracting the sum of the constant terms on the left side of the inequality from both sides of the inequality.

4. $245.60 \leq 9.50h \leq 368.40; 25.9 \leq h \leq 38.8$

The product of time worked and money earned per hour must lie between $245.60 and $368.40. To solve the inequality, divide each side of the inequality by the coefficient of the variable.

5. \{w \mid w > -5\} \hspace{1cm} Remember – whenever you multiply or divide by a negative, you must flip the inequality symbol.

Example for 6 – 7.

**EXAMPLE 3** Solve and Graph a Union

Solve $-2m + 7 \leq 13 \text{ or } 5m + 12 > 37$. Then graph the solution set.

$-2m + 7 \leq 13$ \hspace{1cm} or \hspace{1cm} $5m + 12 > 37$

$-2m + 7 - 7 \leq 13 - 7$ \hspace{1cm} Subtract.

$-2m \leq 6$ \hspace{1cm} Simplify.

$\frac{-2m}{-2} \geq \frac{6}{-2}$ \hspace{1cm} Divide.

$m \geq -3$ \hspace{1cm} Simplify.

$\begin{array}{c}
\text{Graph } m \geq -3. \\
\text{Graph } m > 5. \\
\text{Find the union.}
\end{array}$

Notice that the graph of $m \geq -3$ contains every point in the graph of $m > 5$. So, the union is the graph of $m \geq -3$. The solution set is \{m \mid m \geq -3\}.

6. \{w \mid w \text{ is a real number}\}

7. \{x \mid x > 2\}
8. \( y = \frac{7}{3}x + 1; \ \frac{7}{3} \)

Find the slope of the line with the slope formula. Find the \( y \)-intercept by replacing \( x \) and \( y \) with the given point and \( m \) with the slope in the slope-intercept form. Solve for \( b \). Write the equation in slope-intercept form using the given \( m \) and the calculated \( b \).

9. \( y - 2 = 4(x + 1); \ y = 4x + 6; \ 4x - y = -6 \)

The linear equation \( y - y_1 = m(x - x_1) \) is written in point-slope form, where \((x_1, y_1)\) is a given point on a nonvertical line and \( m \) is the slope of the line.

Given an equation in point-slope form, solve the equation for \( y \) to find the equation in slope-intercept form.

The linear equation in standard form is given as \( Ax + By = C \), where \( A \), \( B \), and \( C \) are constants. Use Addition and Subtraction Properties of Equality to rewrite the equation in standard form.

10. No; the slopes are 4 and \( \frac{1}{4} \).

Two nonvertical lines are perpendicular if the slopes are opposite reciprocals of each other.

11. \( y = -4x + 4 \)

Two nonvertical lines are parallel if they have the same slope. Use the given point with the slope of the parallel line in the point-slope form. Then change to the slope-intercept form.

12. 8 saltwater fish, 7 freshwater fish

\[ x + y = 15 \]
\[ 2x + y = 23 \]
Eliminate one variable by subtracting the two equations. Solve for \( x \) and then substitute that value into one of the equations to find the value of \( y \).

13. Speed of train \( A \): 50mph, Speed of train \( B \): 30mph

\[ x = y + 20 \]
\[ 3x + 3y = 240 \]
Substitute \( y + 20 \) for \( x \) in the second equation and solve for \( y \). Substitute that value into the first equation and solve for \( x \).

14. 10 dimes and 15 quarters

\[ x + y = 25 \]
\[ 0.10x + 0.25y = 4.75 \]
Substitute \( 25 - x \) for \( y \) in the second equation and solve for \( x \). Substitute that value into the first equation and solve for \( y \).

15. 5, 7

\[ 5x + y = 32 \]
\[ 3x - y = 8 \]
Eliminate one variable by adding the two equations. Solve for \( x \) and then substitute that value into one of the equations to find the value of \( y \).

16. \( \left( 7, -3 \right) \)

Eliminate the \( x \) terms by first multiplying the top equation by 3 and the bottom one by 2 and then subtracting the two equations. Solve for \( y \) and then substitute that value into one of the equations to find the value of \( x \).

17. \[ d = t(r - c), \quad d = t(r + c); \quad t = \frac{d}{r - c}, \quad t = \frac{d}{r + c} \]

Use the formula, \( d = rt \), to find the equation for the distance traveled by the boat downstream with the current and upstream with the current.

18. 6.48 mph

\[ 4x + 4y = 33 \]

\[ 7x - 7y = 33 \]

Set up problem in same way as number 17. Eliminate the \( x \) terms by first multiplying the top equation by 7 and the bottom one by 4 and then subtracting the two equations. Solve for \( y \) and then substitute that value into one of the equations to find the value of \( x \).

19. About 42.4 minutes

Time taken to weed the garden if Laura and her husband worked together is \( \frac{1}{2} \) h.

20. 3 h

Time taken to empty a full tank if both pipes are open is \( 1 - \frac{1}{2} \) h.

21.

22. substitution; (3, 2)

23. elimination with subtraction; (2, -7)

24. \( x + 17 \)

Length of \( PQ \) is the sum of the length of the garden and the width of the walkway.

25. 16.5 ft by 25.5 ft

The total area of the walkway is 400 square feet, which is the difference between the area of garden along with walkway and the area of garden. Solve for \( x \) to find the dimensions of the garden.

26. \( 3x^2 + 1 \)

The perimeter of a triangle is the sum of the three sides. Group like terms together. Subtract like terms, making sure you subtract negatives (add). The power stays the same.
Rules for exponents:

For any nonzero real numbers $a$ and $b$ and any integers $m$, $n$, and $p$, the following are true.

**Multiplying Monomials** *(Lesson 7-1)*
- Product of Powers: $a^m \cdot a^n = a^{m+n}$
- Power of a Power: $(a^m)^n = a^{mn}$
- Power of a Product: $(ab)^m = a^m b^m$

**Dividing Monomials** *(Lesson 7-2)*
- Quotient of Powers: $\frac{a^m}{a^p} = a^{m-p}$
- Power of a Quotient: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
- Zero Exponent: $a^0 = 1$
- Negative Exponent: $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$

27. $-6a^3 b^8$
28. $10a^4 b^8$
29. $\frac{d^7}{285}$
30. $3r^5 t$

Example for 31.

**Example 4** Divide with Scientific Notation

Evaluate $\frac{3.066 \times 10^8}{7.3 \times 10^3}$. Express the result in both scientific notation and standard form.

$\frac{3.066 \times 10^8}{7.3 \times 10^3} = \frac{(3.066) \times 10^8}{(7.3) \times 10^3}$

$= 0.42 \times 10^5$

$= 4.2 \times 10^4$

$= 42,000$

31. $2 \times 10^3$, .002
Example for 32–33.

### Example 3

**Standard Form of a Polynomial**

Write each polynomial in standard form. Identify the leading coefficient.

**a.** \(3x^3 + 4x^5 - 7x\)

**Step 1** Find the degree of each term.

<table>
<thead>
<tr>
<th>Degree</th>
<th>3</th>
<th>5</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polynomial</td>
<td>(3x^3 + 4x^5 - 7x)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Step 2** Write the terms in descending order: \(4x^5 + 3x^3 - 7x\).

The leading coefficient is 4.

**b.** \(5y^3 - 2y^4 - 6y^3\)

**Step 1** Degree:

<table>
<thead>
<tr>
<th>Degree</th>
<th>3</th>
<th>4</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polynomial</td>
<td>(5y^3 - 2y^4 - 6y^3)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Step 2** \(-2y^4 - 6y^3 + 5y^3 - 9\) The leading coefficient is -2.

32. 5
33. \(x^3 + 3x^2 - x - 3; 1\)
34. \(6x^4y^2 - 15x^3y^3 + 24x^4y^4\)
35. \(6n^3 + n^2 - 10n + 3\)
36. \(25y^2 + 60y + 36\)

Examples for 37–48.

### Example 4

Solve \(x^2 - 6x = 0\). Check your solutions.

Write the equation so that it is of the form \(ab = 0\).

\[x^2 - 6x = 0\] \hspace{2cm} **Original equation**

\[x(x - 6) = 0\] \hspace{2cm} **Factor by using the GCF.**

\[x = 0 \text{ or } x - 6 = 0\] \hspace{2cm} **Zero Product Property**

\[x = 6\] \hspace{2cm} **Solve.**

The roots are 0 and 6. Check by substituting 0 and 6 for \(x\) in the original equation.

### Example 5

Factor \(x^2 + 10x + 21\)

\(b = 10\) and \(c = 21\), so \(m + p\) is positive and \(mp\) is positive. Therefore, \(m\) and \(p\) must both be positive. List the positive factors of 21, and look for the pair of factors with a sum of 10.

<table>
<thead>
<tr>
<th>Factors of 21</th>
<th>Sum of 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 21</td>
<td>22</td>
</tr>
<tr>
<td>3, 7</td>
<td>10</td>
</tr>
</tbody>
</table>

The correct factors are 3 and 7.

\(x^2 + 10x + 21 = (x + m)(x + p)\) \hspace{2cm} **Write the pattern.**

\(m = 3\) and \(p = 7\)

\(= (x + 3)(x + 7)\)
EXAMPLE 6

Factor $12a^2 + 17a + 6$

$a = 12, b = 17, c = 6$. Since $b$ is positive, $m + p$ is positive. Since $c$ is positive, $mp$ is positive. So, $m$ and $p$ are both positive. List the factors of 12(6) or 72, where both factors are positive.

<table>
<thead>
<tr>
<th>Factors of 72</th>
<th>Sum of 17</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 72</td>
<td>73</td>
</tr>
<tr>
<td>2, 36</td>
<td>38</td>
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<tr>
<td>3, 24</td>
<td>27</td>
</tr>
<tr>
<td>4, 18</td>
<td>22</td>
</tr>
<tr>
<td>6, 12</td>
<td>18</td>
</tr>
<tr>
<td>8, 9</td>
<td>17</td>
</tr>
</tbody>
</table>

The correct factors are 8 and 9.

$12a^2 + 17a + 6 = 12a^2 + ma + pa + 6$

$= 12a^2 + 8a + 9a + 6$

$= (12a^2 + 8a) + (9a + 6)$

$= 4a(3a + 2) + 3(3a + 2)$

$= (3a + 2)(4a + 3)$

So, $12a^2 + 17a + 6 = (3a + 2)(4a + 3)$.

EXAMPLE 7

Solve $x^2 - 4 = 12$ by factoring.

1. $x^2 - 4 = 12$ Original equation
2. $x^2 - 16 = 0$ Subtract 12 from each side.
3. $(x + 4)(x - 4) = 0$ Factor the difference of squares.
4. $x + 4 = 0$ or $x - 4 = 0$ Zero Product Property
5. $x = -4$ or $x = 4$ Solve each equation.
6. The solutions are $-4$ and $4$.

37. $2x^3yz(5 - 11xy)$
38. $(y - 2)(2x + 3)$
39. $(m + 14)(m - 2)$
40. $(5t - 3)(t + 4)$
41. $2(3p - 4)(p - 2)$
42. $(7a + 13)(7a - 13)$
43. $3x^2(x + 5)(x - 5)$
44. $(9c + 4)^2$
45. prime
46. $\left\{0, \frac{2}{3}\right\}$
47. $\{-9, 5\}$
48. $\left\{-1, \frac{1}{3}\right\}$
Examples for 49 – 51.

**Example 2: Use the Quadratic Formula**

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

a. \(3x^2 + 5x - 12 = 0\)

For this equation, \(a = 3, \ b = 5, \) and \(c = -12.\)

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
= \frac{-5 \pm \sqrt{(-5)^2 - 4(3)(-12)}}{2(3)}
\]

\[
= \frac{-5 \pm \sqrt{25 + 144}}{6}
\]

\[
= \frac{-5 \pm \sqrt{169}}{6} \quad \text{or} \quad \frac{-5 \pm 13}{6}
\]

\[
x = \frac{-5 - 13}{6} \quad \text{or} \quad x = \frac{-5 + 13}{6}
\]

\[
= -3 \quad \text{or} \quad \frac{4}{3}
\]

The solutions are \(-3\) and \(\frac{4}{3}\).

b. \(10x^2 - 5x = 25\)

**Step 1**  Rewrite the equation in standard form.

\[
10x^2 - 5x = 25
\]

\[
10x^2 - 5x - 25 = 0
\]

Original equation

Subtract 25 from each side.

**Step 2**  Apply the Quadratic Formula.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(10)(-25)}}{2(10)}
\]

\[
= \frac{5 \pm \sqrt{25 + 1000}}{20}
\]

\[
= \frac{5 \pm \sqrt{1025}}{20}
\]

\[
= \frac{5 - \sqrt{1025}}{20} \quad \text{or} \quad \frac{5 + \sqrt{1025}}{20}
\]

\[
\approx -1.4 \quad \text{or} \quad 1.9
\]

The solutions are about \(-1.4\) and \(1.9\).
54. Mark the points \((-2, 0), (0, 4),\text{ and } (2, 0)\) on the graph and draw a smooth curve.

55. One root lies between \(-3\) and \(-2\) and other root lies between \(-2\) and \(-1\).

minimum point: \((2, -1)\)

Roots are the x-intercept and the minimum point is the vertex of the function.

Example for 56.

**Identify Characteristics from Functions**

Find the vertex, the equation of the axis of symmetry, and the y-intercept of each function.

a. \(y = 2x^2 + 4x - 3\)

- \(x = -\frac{b}{2a}\) Formula for the equation of the axis of symmetry
- \(x = -\frac{4}{2 	imes 2} = -1\) \(\sigma = 2\text{ and } b = 4\)
- \(x = -1\) Simplify.

The equation for the axis of symmetry is \(x = -1\).

To find the vertex, use the value you found for the axis of symmetry as the \(x\)-coordinate of the vertex. To find the \(y\)-coordinate, substitute that value for \(x\) in the original equation.

\[
y = 2x^2 + 4x - 3
\]

\[
= 2(-1)^2 + 4(-1) - 3 \quad x = -1
\]

\[
= -5
\]

Simplify.

The vertex is at \((-1, -5)\).
The \(y\)-intercept always occurs at \((0, c)\). So, the \(y\)-intercept is \(-3\).

56. \(x = 1; (1, -3)\); maximum

Examples for 57 – 60.
Substitute the given values in the Distance Formula. Use the Product Property of square roots. Then simplify the result.

62. \( b = 0 \) or \( 6 \)

Substitute the given values in the Distance Formula. Evaluate the squares. Solve for the missing variable.

63. \( 264 + 24\sqrt{21} \text{ in.}^2, 8\sqrt{3} + 24\sqrt{7} \text{ in.} \)

The area of a square is \( s^2 \), where \( s \) is the side of the square and the formula for the perimeter of a rectangle is \( 4s \), where \( s \) is the side of the square. Replace the variables in the formula with the given values. Use the Product Property of square roots. Then simplify the result.

64. \( 24 \text{ cm}^2 \)

Find the height of the triangle by the Pythagorean theorem \( (a^2 + b^2 = c^2) \). The area of a triangle is \( \frac{1}{2} \times \text{base} \times \text{height} \). Replace the variables in the formula and solve.

65. \( 9 \text{ in.} \)

Since the two triangles are similar, the measures of their corresponding sides are proportional. You can use the ratios of the corresponding sides of the triangles to determine the missing sides.

66. \( (3, 2) \)
Examples for 67 – 78.

**EXAMPLE 2**

State the excluded value for the function

\[ y = \frac{1}{4x + 16}. \]

Set the denominator equal to zero.

\[ 4x + 16 = 0 \]

Subtract 16 from each side.

\[ 4x = -16 \]

Simplify.

\[ x = -4 \]

Divide each side by 4.

**EXAMPLE 3**

Simplify \( \frac{a^2 - 7a + 12}{a^2 - 13a + 36} \).

Factor and simplify.

\[ \frac{a^2 - 7a + 12}{a^2 - 13a + 36} = \frac{(a - 3)(a - 4)}{(a - 9)(a - 4)} \]

Factor.

\[ \frac{a - 3}{a - 9} \]

Simplify.

**EXAMPLE 5**

Add Rational Expressions with Unlike Denominators

Find \( \frac{3t + 2}{t^2 - 2t - 3} + \frac{t + 1}{t - 3} \).

Find the LCD. Since \( t^2 - 2t - 3 = (t - 3)(t + 1) \), the LCD is \( t - 3 \).

\[ \frac{3t + 2}{t^2 - 2t - 3} + \frac{t + 1}{t - 3} = \frac{3t + 2}{(t - 3)(t + 1)} + \frac{t + 1}{t - 3} \]

Factor \( t^2 - 2t - 3 \).

\[ = \frac{3t + 2}{(t - 3)(t + 1)} + \frac{t + 1}{t - 3} \]

Write \( \frac{t + 1}{t - 3} \) using the LCD.

\[ = \frac{3t + 2}{(t - 3)(t + 1)} + \frac{t + 1}{(t - 3)(t + 1)} \]

Simplify.

\[ = \frac{3t + 2 + t + 1}{(t - 3)(t + 1)} \]

Add the numerators.

\[ = \frac{t^2 + 3t + 3}{(t - 3)(t + 1)} \]

Simplify.

67. \( \frac{k + 5}{k - 1}, 1, 3 \)

68. \( 2(y - 3) \)

69. \( \frac{1}{r + 1} \)

70. \( \frac{x + 1}{4x} \)

71. \( \frac{6(x + 7)}{7(x - 5)} \)

72. \( \frac{4a^2b - 5b^2}{3} - 5ab \)

73. \( \frac{8x + 4}{r - 5} \)

74. \( \frac{4}{\sqrt[1]{2}} \)

75. \( \frac{2a + 25}{a^2 - 36} \)

76. \( \frac{25x^2 + 16}{5n - 4} \)

77. \( \frac{-9y + 20}{5y(\gamma - 7)} \)
Example for 79–80.

**EXAMPLE 3** Exogenous Solutions

Solve \( \frac{2n}{n - 5} + \frac{4n - 30}{n - 5} = 5 \). State any exogenous solutions.

\[
\frac{2n}{n - 5} + \frac{4n - 30}{n - 5} = 5 \quad \quad \quad \text{Original equation}
\]

\[
(n - 5) \left( \frac{2n}{n - 5} + \frac{4n - 30}{n - 5} \right) = (n - 5)5 \quad \quad \quad \text{Multiply each side by the LCD, } n - 5.
\]

\[
\left( \frac{1}{1}, \frac{2n}{n - 5} \right) + \left( \frac{1}{1}, \frac{4n - 30}{n - 5} \right) = (n - 5)5 \quad \quad \quad \text{Distributive Property}
\]

\[
2n + 4n - 30 = 5n - 25 \quad \quad \quad \text{Simplify.}
\]

\[
6n - 30 = 5n - 25 \quad \quad \quad \text{Add like terms.}
\]

\[
6n - 5n - 30 = 5n - 5n - 25 \quad \quad \quad \text{Subtract 5n from each side.}
\]

\[
n - 30 = -25 \quad \quad \quad \text{Simplify.}
\]

\[
n - 30 + 30 = -25 + 30 \quad \quad \quad \text{Add 30 to each side.}
\]

\[
n = 5 \quad \quad \quad \text{Simplify.}
\]

Since \( n = 5 \) results in a zero in the denominator of the original equation, it is an exogenous solution. So, the equation has no solution.

79. \(-4\)

80. \(-5\); exogenous 3

Example for 81.

**EXAMPLE 2** Solve Absolute Value Equations

Solve each equation. Then graph the solution set.

a. \(|f + 5| = 17\)

\[|f + 5| = 17 \quad \quad \quad \text{Original equation}
\]

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f + 5 = 17)</td>
<td>(f + 5 = -17)</td>
</tr>
<tr>
<td>(f + 5 - 5 = 17 - 5)</td>
<td>(f + 5 - 5 = -17 - 5)</td>
</tr>
<tr>
<td>(f = 12)</td>
<td>(f = -22)</td>
</tr>
</tbody>
</table>

b. \(|b - 1| = -3\)

\(|b - 1| = -3\) means the distance between \( b \) and 1 is -3. Since distance cannot be negative, the solution is the empty set \( \emptyset \).

81. \(x = -4\) or \(x = 12\)
Examples for 83-84.

**EXAMPLE 1** Solve Absolute Value Inequalities (<)

Solve each inequality. Then graph the solution set.

a. \(|m + 2| < 11\)

   Rewrite \(|m + 2| < 11\) for Case 1 and Case 2.

   **Case 1** \(m + 2\) is nonnegative. and **Case 2** \(m + 2\) is negative.

   \[
   \begin{align*}
   m + 2 &< 11 \\
   m + 2 - 2 &< 11 - 2 \\
   m &< 9 \\
   \end{align*}
   \]

   \[
   \begin{align*}
   -(m + 2) &< 11 \\
   m + 2 &> -11 \\
   m + 2 - 2 &> -11 - 2 \\
   m &> -13 \\
   \end{align*}
   \]

   So, \(m < 9\) and \(m > -13\). The solution set is \(|m| \sim -13 < m < 9\).

b. \(|y - 1| < -2\)

   \(|y - 1|\) cannot be negative. So it is not possible for \(|y - 1|\) to be less than \(-2\).

   Therefore, there is no solution, and the solution set is the empty set, \(\emptyset\).
EXAMPLE 3  Solve Absolute Value Inequalities (>)

Solve $|3n + 6| \geq 12$. Then graph the solution set.

Rewrite $|3n + 6| \geq 12$ for Case 1 or Case 2.

**Case 1** $3n + 6$ is nonnegative.  

\[
3n + 6 \geq 12 \\
3n \geq 6 \\
n \geq 2
\]

**Case 2** $3n + 6$ is negative.  

\[
-(3n + 6) \geq 12 \\
3n + 6 \leq -12 \\
3n \leq -18 \\
n \leq -6
\]

So, $n \geq 2$ or $n \leq -6$. The solution set is \( \{ n \mid n \geq 2 \text{ or } n \leq -6 \} \).

83. $d < -9$ or $d > 7$

84. $-3 \leq x \leq 6$